

THE LIGHT FRONT GAUGE PROPAGATOR: THE STATUS QUO

A. T. Suzuki^a and J.H.O. Sales^b

ABSTRACT. At the classical level, the inverse differential operator for the quadratic term in the gauge field Lagrangian density fixed in the light front through the multiplier $(n \cdot A)^2$ yields the standard two term propagator with single unphysical pole of the type $(k \cdot n)^{-1}$. Upon canonical quantization on the light-front, there emerges a third term of the form $(k^2 n^\mu n^\nu)(k \cdot n)^{-2}$. This third term in the propagator has traditionally been dropped on the grounds that is exactly cancelled by the “instantaneous” term in the interaction Hamiltonian in the light-front. Our aim in this work is not to discuss which of the propagators is the correct one, but rather to present at the classical level, the gauge fixing conditions that can lead to the three-term propagator. It is revealed that this can only be accomplished via two coupled gauge fixing conditions, namely $n \cdot A = 0 = \partial \cdot A$. This means that the propagator thus obtained is doubly transversal.

1. INTRODUCTION

As early as 1970 with J.B.Kogut and D.E.Soper [1] and a little later with E.Tomboulis in 1973 [2], light-front gauge propagator for Abelian and Non-Abelian gauge fields derived via canonical quantization was known to have a (third) term proportional to $(k^2 n_\mu n_\nu)(k \cdot n)^{-2}$. According to the latter, “*The third term represents an instantaneous “Coulomb”-type interaction.*” Moreover, he (see also [3]) showed then that “*We will now show that all graphs representing the Coulomb term ... precisely cancel the contributions from the last term of the propagator ... so that we are left with an effective interaction Hamiltonian density ... i.e, the two usual vertices, and a propagator...*” where the so-called *effective* propagator is the traditional two-term light-front propagator:

$$(1.1) \quad G^{\mu\nu ab}(k) = -\frac{\delta^{ab}}{k^2 + i\varepsilon} \left[g^{\mu\nu} - \frac{k^\mu n^\nu + k^\nu n^\mu}{k \cdot n} \right]$$

More recently, P.P.Srivastava and S.J.Brodsky [4] rederived the three-term “doubly transverse gauge propagator”

$$(1.2) \quad G^{\mu\nu ab}(k) = -i \frac{\delta^{ab}}{k^2 + i\varepsilon} \left[g^{\mu\nu} - \frac{k^\mu n^\nu + k^\nu n^\mu}{k \cdot n} + \frac{k^2 n^\mu n^\nu}{(k \cdot n)^2} \right],$$

for QCD in the framework of Dyson-Wick S-matrix expansion with a *BRS symmetric* Lagrangian density. They then apply the Dirac method of implementing the field constraints (first and second classes) to obtain the interaction Hamiltonian

from which the canonical quantization is performed via correspondence principle between Poisson brackets and Dirac commutators for the field operators. Their derivation clearly shows the conspicuous instantaneous interaction terms (the so-called tree-level *seagull* diagram terms) present in the interaction Hamiltonian in the light-front. Their explicit calculations for the electron-muon scattering in the Abelian QED theory in the light-front as well as the one-loop β -function for the non-Abelian Yang-Mills fields, with gluon vacuum polarization tensor, three-point vertex functions and gluon self-energy corrections from the quark loop, show us the subtle cancellations that come to play a crucial role into the game of light-front renormalization program with instantaneous interaction terms in the Hamiltonian and the third term of the gluon propagator.

On the other hand, if one uses the classical approach of inverting the differential operator sandwiched between the quadratic term in the Lagrangian density plus the gauge fixing term of the form $(n \cdot A)^2$ in order to obtain the gauge field propagator, the result is straightforwardly given by (1.1). There is no way - classically - to arrive at (1.2) with only the gauge fixing Lagrangian of the form $(n \cdot A)^2$. This means that, as it stands, there is an anomaly between the classical and the quantum propagator.

2. CLASSICALLY DEDUCIBLE THREE-TERM L.F. PROPAGATOR

Since at the classical level we just look for the inverse operator sandwiched between the quadratic term in the Lagrangian density plus the gauge fixing term, in order to get a three-term propagator we need to incorporate not only the usual $n \cdot A = 0$ condition into the gauge fixing part, but couple it to the Lorentz condition $\partial \cdot A = 0$. The reason why we need the latter condition becomes clear when one understands that the Lorentzian condition coupled to the former gauge condition is nothing more than the constraint equation for the unphysical field component A^- , which is not a dynamical variable in the light-front formalism. Note that this does not remove too many degrees of freedom from the gauge fields as one would naively think, but that both of those two are in fact necessary to completely fix the gauge in the light-front with no residual gauge freedom left. In a recent work, we have shown how this can be accomplished [5] via considering one Lagrange multiplier of the form $(n \cdot A)(\partial \cdot A)/\alpha$, where α is the single gauge fixing parameter.

In this work we show that the one gauge fixing term above referred to can be generalized to a two term general gauge fixing term of the form $(n \cdot A)^2/\alpha + (\partial \cdot A)^2/\beta$, where now α and β are two independent gauge fixing parameters, yielding the same result, namely, the three-term, “doubly transverse propagator” (1.2). .

The Lagrangian density for the vector gauge field (for simplicity we consider an Abelian case) is given by

$$(2.1) \quad \mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\beta}(\partial_\mu A^\mu)^2 - \frac{1}{2\alpha}(n_\mu A^\mu)^2 = \mathcal{L}_E + \mathcal{L}_{GF}$$

By partial integration and considering that terms which bear a total derivative don't contribute and that surface terms vanish since $\lim_{x \rightarrow \infty} A^\mu(x) = 0$, we have

$$(2.2) \quad \mathcal{L}_E = \frac{1}{2}A^\mu (\partial^2 g_{\mu\nu} - \partial_\mu \partial_\nu) A^\nu$$

and

$$(2.3) \quad \mathcal{L}_{GF} = -\frac{1}{2\beta}\partial_\mu A^\mu \partial_\nu A^\nu - \frac{1}{2\alpha}n_\mu A^\mu n_\nu A^\nu$$

$$(2.4) \quad = \frac{1}{2\beta}A^\mu \partial_\mu \partial_\nu A^\nu - \frac{1}{2\alpha}A^\mu n_\mu n_\nu A^\nu$$

so that

$$(2.5) \quad \mathcal{L} = \frac{1}{2}A^\mu \left(\partial^2 g_{\mu\nu} - \partial_\mu \partial_\nu + \frac{1}{\beta}\partial_\mu \partial_\nu - \frac{1}{\alpha}n_\mu n_\nu \right) A^\nu$$

To find the gauge field propagator we need to find the inverse of the operator between parenthesis in (2.5). That differential operator in momentum space is given by:

$$(2.6) \quad O_{\mu\nu} = -k^2 g_{\mu\nu} + k_\mu k_\nu - \theta k_\mu k_\nu - \lambda n_\mu n_\nu,$$

where $\theta = \beta^{-1}$ and $\lambda = \alpha^{-1}$, so that the propagator of the field, which we call $G^{\mu\nu}(k)$, must satisfy the following equation:

$$(2.7) \quad O_{\mu\nu} G^{\nu\lambda}(k) = \delta_\mu^\lambda$$

$G^{\nu\lambda}(k)$ can now be constructed from the most general tensor structure that can be defined, i.e., all the possible linear combinations of the tensor elements that composes it [6]:

$$(2.8) \quad G^{\mu\nu}(k) = g^{\mu\nu}A + k^\mu k^\nu B + k^\mu n^\nu C + n^\mu k^\nu D + k^\mu m^\nu E +$$

$$(2.9) \quad + m^\mu k^\nu F + n^\mu n^\nu G + m^\mu m^\nu H + n^\mu m^\nu I + m^\mu n^\nu J$$

where m^μ is the light-like vector dual to the n^μ , and $A, B, C, D, E, F, G, H, I$ and J are coefficients that must be determined in such a way as to satisfy (2.7). Of course, it is immediately clear that since (2.5) does not contain any external light-like vector m_μ , the coefficients $E = F = H = I = J = 0$ straightaway. Then, we have

$$(2.10) \quad A = -(k^2)^{-1}$$

$$(2.11) \quad (k \cdot n)(1 - \theta)G - \theta k^2 D = 0$$

$$(2.12) \quad (-k - \lambda n^2)G - \lambda(k \cdot n)D - \lambda A = 0$$

$$(2.13) \quad -(k^2 + \lambda n^2)C - \lambda(k \cdot n)B = 0$$

$$(2.14) \quad (1 - \theta)A - \theta k^2 B + (1 - \theta)(k \cdot n)C = 0$$

From (2.11) we have

$$(2.15) \quad G = \frac{k^2}{(k \cdot n)(\beta - 1)} D$$

which inserted into (2.12) yields

$$(2.16) \quad D = \frac{-(k \cdot n)(\beta - 1)}{(\alpha k^2 + n^2)k^2 + (k \cdot n)^2(\beta - 1)} A$$

From (2.13) and (2.14) we obtain

$$(2.17) \quad B = \frac{-(\alpha k^2 + n^2)}{k \cdot n} C$$

and

$$(2.18) \quad C = \frac{-(\beta - 1)(k \cdot n)}{(\alpha k^2 + n^2)k^2 + (k \cdot n)^2(\beta - 1)} A = D$$

In the light-front, $n^2 = 0$ and taking the limits $\alpha, \beta \rightarrow 0$, we have

$$(2.19) \quad A = \frac{-1}{k^2}$$

$$(2.20) \quad B = 0$$

$$(2.21) \quad C = D = \frac{1}{k^2(k \cdot n)}$$

$$(2.22) \quad G = \frac{-1}{(k \cdot n)^2}$$

Therefore, the relevant propagator in the light-front gauge is:

$$(2.23) \quad G^{\mu\nu}(k) = -\frac{1}{k^2} \left\{ g^{\mu\nu} - \frac{k^\mu n^\nu + n^\mu k^\nu}{k \cdot n} + \frac{n^\mu n^\nu}{(k \cdot n)^2} k^2 \right\},$$

which has the outstanding third term commonly referred to as *contact term*. This procedure eliminates the problem of the existing anomaly between the classical and quantum derivations for the light-front gauge propagator.

3. Conclusions

We have shown that at the classical level we can introduce two Lagrange multipliers in the Lagrangian density that is consistent with gauge fixing in the light-front. No excess degrees of freedom is eliminated with this formalism since the coupled conditions are such that the Lorentzian condition yields nothing more than a constraint equation for the non-dynamical variable A^- . This means that with both these coupled conditions, the ensuing gauge field is entirely defined in its dynamical transverse degrees of freedom, i.e., there is no residual gauge freedom left. Moreover, the consistency of the procedure is enhanced by the fact that the propagator thus deduced is the same as the one obtained via canonical quantization in the light-front (no anomaly). The reason why this is so can be seen from the fact that since $n \cdot A = 0 = \partial \cdot A$ it follows that $(n \cdot A + \partial \cdot A)^2 = 0$ from which $(n \cdot A)^2 + (\partial \cdot A)^2 = -2(n \cdot A)(\partial \cdot A)$ [7]. Of course, no attempt is here made to discuss whether one should work in the usual two-term reduced (or sometimes called effective) propagator or in the three-term (which we call full) propagator. Our aim here was solely to solve the anomaly problem and pinpoint a solution to the classical problem when confronted with the quantum derivation.

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^{a,b}INSTITUTO DE FÍSICA TEÓRICA-UNESP, 01405-900, SÃO PAULO, BRAZIL.